Two Stages Signal Strength Difference Localization Algorithm using SDP Relaxation

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Abstract—We present a novel two stages signal strength difference (TS-SSD) localization algorithm in this letter. A new model using TS-SSD technique is derived to eliminate the effects of path loss exponent and unknown transmit power. And a total least squares (TLS) solution is given to estimate the distances between anchor and target nodes. Then a low-rank matrix completion framework is established to estimate the distances among all the unknown target nodes. After obtaining all distance measurements, a simple but robust approach based on Semidefinite Programming (SDP) relaxation is applied to estimate the location of target nodes. Compared with some existing algorithms, the proposed approach has three-fold merits: firstly, it locates multiple targets using less anchor nodes simultaneously; secondly, it shows more robustness to path model error and noise level; thirdly, it has higher localization accuracy than some existing methods. Simulation results illustrate the superiority of our proposed algorithm.

Index Terms—Localization; two stages signal strength difference (TS-SSD); matrix completion; SDP;

I. INTRODUCTION

Localization is one of the essential modules of many mobile wireless applications [1, 2]. Because Received Signal Strength (RSS) measurement can be easily obtained on most off-the-shelf equipments, such as WiFi- or ZigBee compatible devices, a majority of previous localization approaches with RSS as a metric for location determination are proposed in the past two decades.

The accuracy of some existing RSS-based localization algorithms depends highly on the knowledge of relevant parameters, including transmit power, path loss exponent, and noise level [1–3]. Unfortunately, securing these parameters accurately in unknown environment is time intensive. As a consequence, two kinds of methods are presented to overcome the drawback. One tries to estimate the unknown node locations directly without knowing the path loss exponent [4, 5]. The performance of these methods is good in high SNR case, but they cannot estimate multiple targets simultaneously. The other is fingerprinting based method which does not need to estimate the nuisance parameters [6]. It can improve the accuracy of localization to some extent, but the fingerprinting database building process is a time consuming job. Signal strength difference (SSD) technique is a good way to mitigate the effect of unknown transmit power [7] and it improves the performance of localization greatly but it still belongs to fingerprinting based localization framework.

Instead of estimating the target locations from the noisy RSS measurements directly, we recover the distance matrix among all nodes from noisy RSS measurements firstly, and the locations of target nodes are estimate from the recovered distance matrix using SDP framework. The main steps of our proposed algorithm are as follows:

a) derive a new model from the conventional path loss model using two stages SSD approach;
b) a TLS solution is used to calculate the distances between anchor nodes and target nodes;
c) estimate the distances among all target nodes using low-rank matrix completion method;
d) target locations are estimated from the distance matrix among all nodes using SDP relaxation.

Compared with some existing methods, such as multidimensional scaling (MDS) method [8], and distributed weighted-multidimensional scaling (dwMDS) method [9], the merits of our method are threefold: firstly, it locates multiple targets using less anchor nodes simultaneously; secondly, it shows more robustness to path model error and noise level; thirdly, it has higher localization accuracy than some existing methods.

A. The proposed algorithm

B. RSS measurements model and distance matrix

Given \( M (M \geq 3) \) anchor nodes (We suppose that at least three of \( M \) can communicate with each other) in a wireless localization system with locations \( O = \{o_1, o_2, \ldots, o_M\} \), and \( N \) target nodes whose locations \( X = \{x_1, x_2, \ldots, x_N\} \) are to be determined.

Let \( d_{i,j} = \|x_i - x_j\|_2 \) denote the distance between target nodes \( i \) and \( j \), \( d_{k,i} = \|o_k - x_i\|_2 \) denote the distance between anchor node \( k \) and target node \( i \), and \( d_{k,l} = \|o_k - o_l\|_2 \) denote the distance between anchor nodes \( k \) and \( l \), where \( \|\cdot\|_2 \) denotes the \( \ell_2 \)-norm. The nonnegative and symmetric distance matrix among all nodes with zero diagonal can be expressed as

\[
D = \begin{bmatrix}
D_{aa} & D_{au} \\
D_{ua} & D_u
\end{bmatrix},
\]

(1)

where \( D_{aa} = (d_{i,j}^2), D_{au} = D_{ua}^T = (d_{i,k}^2), \) and \( D_u = (d_{k,l}^2). \)
Suppose that $P_{i,j}$ and $P_i$ are the RSS measurements (measured at anchor node $i$) at an arbitrary distance $d_{i,j}$ and a close-in reference distance $d_0$ from the node $j$, respectively. From the log-normal shadowing model [1], we have

$$ P_{i,j} = P_0 - 10 \log_{10} \frac{d_{i,j}}{d_0} + n_i, \quad (2) $$

where $\beta$ denotes the path loss exponent, $n_i$ is empirically modeled as a lognormal random variable with zero mean and variance $\sigma^2_i$. The problem of localization using RSS measurements is to estimate the locations of target nodes $X$ from $P_{i,j}$. Obviously, if we can estimate $D$ from the noisy RSS measurements, the location of target nodes can be estimated by SDP technique.

C. The estimates of $D_{a}$ and $D_{u}$ using two stages SSD (TS-SSD) method

If we neglect the noise elements in (2), we have

$$ \bar{P}_{i,j} = P_0 - 10 \log_{10} \frac{d_{i,j}}{d_0}. \quad (3) $$

Let $\bar{P}_{m,i}$ be the RSS between the $m$th and $i$th ($i = 1,2,\ldots,M,i \neq m$) anchor nodes, and $P_{m,j}$ be the RSS between the $m$th anchor and the $j$th ($j = 1,2,\ldots,N$) target node. Take the $m$th anchor node as the reference node, we compute the signal strength difference by subtracting $P_{m,j}$ from $\bar{P}_{m,i}$, yields

$$ \bar{P}_{m,i} - P_{m,j} = -10\beta \log_{10} \frac{z_{m,i}}{z_{m,j}}. \quad (4) $$

Usually, $\beta$ can be assumed a constant in a short period of observation time [4]. Hence, we can compute the signal strength difference for the second time,

$$ \frac{\bar{P}_{m,i} - P_{m,j}}{\bar{P}_{m,k} - P_{m,j}} = \frac{\log_{10} z_{m,i} - \log_{10} z_{m,j}}{\log_{10} z_{m,k} - \log_{10} z_{m,j}}. \quad (5) $$

where $k = 1,2,\ldots,M$ ($k \neq m,k \neq m+1$) is a subset of $i$. If $m = M$, we set $m = 1 = 1$. Let $z_{m,i} = \log_{10} z_{m,i}$ and $z_{m,j} = \log_{10} z_{m,j}$, respectively, (5) can be rewritten as

$$ (\bar{P}_{m,i} - P_{m,j}) z_{m,i} = (\bar{P}_{m,k} - P_{m,j}) z_{m,k} - (\bar{P}_{m,k} - P_{m,j}) z_{m,m+1}. \quad (6) $$

Denoting the $N \times 1$ distance vector $z_{\cdot m} = [z_{m,1}, z_{m,2}, \ldots, z_{m,N}]^T$, (6) can be reformulated as

$$ A_m z_{\cdot m} = b_m, \quad (7) $$

where $A_m \in \mathbb{R}^{Q \times N} (Q = (M - 2) \times N)$ is given by $A_m = \text{diag}(a_1, a_2, \ldots, a_N)$ and all the $a_i = 10 \bar{P}_{m,i+1} - \bar{P}_{m,i}$ are the same vector with dimension $(M - 2) \times 1$. The $Q \times 1$ measurement vector $b_m = [b_1, b_2, \ldots, b_N]^T$ and its $j$th element $b_j = (\bar{P}_{m,j} - P_{m,j}) z_{\cdot m} - (\bar{P}_{m,j} - P_{m,j}) z_{m,m+1}$, where 1 is an $(M - 2) \times 1$ all ones vector. $z_{m} = [z_{m,1}, \ldots, z_{m,k}, \ldots, z_{m,M}]^T$, $k \neq m, m+1$, and $\bar{P}_{m} = [\bar{P}_{m,1}, \ldots, \bar{P}_{m,k}, \ldots, \bar{P}_{m,M}]^T$, $k \neq m, m+1$. Observing $A_m$ is an over-determined matrix, (7) has a least squares (LS) solution. However, the new model is derived under noise free environment, errors exist on both sides of (7). To obtain a more robust solution, we consider

$$ (A_m + E_m) z_{\cdot m} = b_m + e_m, \quad (8) $$

where $E_m$ and $e_m$ are error matrix and error vector of $A_m$ and $b_m$, respectively. Obviously, (8) can be rewritten as

$$ (F_m + G_m) d_m = 0, \quad (9) $$

where $F_m = [-b_m, A_m]$ is an extended matrix, $G_m = [-e_m, E_m]$ is a disturbance matrix, and $d_m = [1, z_{\cdot m}]^T$ is a $(N + 1) \times 1$ vector to be determined.

Let singular value decomposition (SVD) of $F_m$ be $F_m = U \Sigma V^H$, with $U$ and $V$ be the left and right singular vectors, respectively. And $\Sigma$ is a $Q \times (N + 1)$ matrix whose elements are zero except possibly along its main diagonal. These nonnegative diagonal elements are ordered such that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{N+1}$. And the right singular vector which corresponds to these singular values is $V = [v_1, v_2, \ldots, v_{N+1}]$ and its $(N + 1)$th element can be expressed as $v_{N+1} = [v(1, N + 1), v(2, N + 1), \ldots, v(M, N + 1)]^T$.

The TLS solution of (9) can be given by the singular vector which corresponds to the smallest singular value $\sigma_{N+1}$

$$ \hat{z}_{\cdot m}^{TLS} = \frac{1}{v(1, N + 1)} [v(2, N + 1)]. \quad (10) $$

Note that if the number of the smallest singular value is more than one, the approximate TLS solution can be found using some modified smoothing techniques [10], which shows more robustness than the conventional LS solution.

Once $\hat{z}_{\cdot m}$ is obtained, the distances can be given by $\hat{d}_m = 10^{\hat{z}_{\cdot m}^{TLS}}$ and we can build a distance matrix $D_{at}$. The matrix $D_{la}$ is given by $D_{la} = D_{at}^T$.

D. The estimate of $D_{u}$ using low-rank matrix completion

After obtaining $D_{at}$ and $D_{la}$, $D_{ui}$ is the rest part of $D$ to be determined. Supposing $\Omega$ denotes the set of all entries of $D_{at}$, $D_{at}$, and $D_{la}$. The distance matrix recovering problem in (1) can be considered as a low-rank matrix completion (LRMC) problem [11]. It has been proven that the rank of matrix $D$ is 4 for 2D geometry localization case and it has a low-rank property. And $D_{at}$ can be fixed by $D_{at} = D_{ui}D_{la}D_{at}$, where $D_{la}$ denotes pseudoinverse of $D_{at}$. The solver performs well with noiseless $D$ and exactly known $D_{at}$ which are impossible in practice.

The LRMC problem in (1) can be solved by formulating the problem into a rank minimization problem [11–13]. Unfortunately, it is non-convex and NP-hard. Candes [11] creatively showed that the rank minimization problem can be approximately solved by its convex relaxation problem

$$ \begin{array}{ll}
\text{minimize} & \| \tilde{D} \|_* \\
\text{subject to} & \tilde{D}(i,j) = D(i,j), (i,j) \in \Omega,
\end{array} \quad (11) $$

where $\| \cdot \|_*$ represents the Nuclear norm and the number of $\Omega$ obeys

$$ m \geq 2r (M + N) - r^2. \quad (12) $$
where \( r \) is the rank of \( D \) and \( \hat{D} \) is the estimate of \( D \). The minimum value of \( M \) with respect to \( N \) should satisfy (12).

Note that \( D \) is not a positive semidefinite (PSD) matrix, we cannot use LRMC method to recover \( D_{\text{at}} \) directly. Inspired by the Lemma 1 in [13], we can construct a PSD matrix by the interior point method as

\[
\begin{bmatrix}
Y & \hat{D} \\
\hat{D}^T & Z
\end{bmatrix} \geq 0. \tag{13}
\]

Then the rank minimization problem in (11) can be equivalently reformulated as

\[
\begin{align*}
\text{minimize} & \quad \delta \\
\text{subject to} & \quad \text{Tr}(Y) + \text{Tr}(Z) \leq 2\delta \\
& \quad \begin{bmatrix} Y & \hat{D} \\ \hat{D}^T & Z \end{bmatrix} \geq 0 \\
& \quad \hat{D}(i,j) = D(i,j), (i,j) \in \Omega, \quad i,j \in \mathbb{Z} \\
& \quad \Omega \supseteq \{ (i,j) \mid r = 1, (i,j) \in \Omega \}, \quad (14)
\end{align*}
\]

where \( \delta \) is a tolerance variable, \( Y \in \mathbb{R}^{(M+N) \times (M+N)} \) and \( Z \in \mathbb{R}^{(M+N) \times (M+N)} \) are the slack matrices, \( \text{Tr} (\cdot) \) denotes the trace operator.

Furthermore, considering that \( D \) is a symmetric matrix, the problem in (14) can be simplified as

\[
\begin{align*}
\text{minimize} & \quad \delta \\
\text{subject to} & \quad \text{Tr}(Q) \leq \delta \\
& \quad \begin{bmatrix} X & \hat{D} \\ \hat{D}^T & X \end{bmatrix} \geq 0 \\
& \quad \hat{D}(i,j) = D(i,j), (i,j) \in \Omega. \quad (15)
\end{align*}
\]

Obviously, it is a semidefinite program problem which can be efficiently solved by CVX [14]. \( D \) can be estimated by the modified interior point method above and \( D_{\text{at}} \) is given by

\[
D = \begin{bmatrix} D & X \\ X^T & X \end{bmatrix} = \begin{bmatrix} D & X \\ X^T & \hat{X} \end{bmatrix} \begin{bmatrix} I_d & \hat{X} \\ \hat{X}^T & \hat{X} \end{bmatrix} = \begin{bmatrix} I_d & \hat{X} \\ \hat{X}^T & \hat{X} \end{bmatrix} D_{\text{at}} \begin{bmatrix} I_d & \hat{X} \\ \hat{X}^T & \hat{X} \end{bmatrix}. \tag{16}
\]

The location estimates of target nodes \( \hat{X} \) can be easily obtained from above optimization problem.

### II. NUMERICAL RESULTS

Firstly, we investigate the computing accuracy of the \( D_{\text{at}} \). Consider a 2-D geometry of \( M = 4 \) anchors with known coordinates at \((-20, 20), (20, 20), (20, -20), (-20, -20)\), while the unknown positions of three target nodes are \((7.5, 7.5), (3, 7), (15, 15)\). We generate the noisy RSSI values \( P_{i,j} \) using (2) with \( \beta = 2.5, d_0 = 1 \) m and the noise variance, \( \sigma_n^2 \), is assigned with \( \text{SNR} = -10 \log_{10} \sigma_n^2 \). In the conventional method [4], the element of \( \hat{D}_{\text{at}} \) is calculated by \( \hat{d}_{i,j} = d_0 \cdot 10^{-0.1 \cdot \text{SNR}} \), while TS-SSD estimates \( \hat{D}_{\text{at}} \) using (10).

The estimated error is calculated by \( \text{Error} = \frac{\| \hat{D}_{\text{at}} - D_{\text{at}} \|^2}{\parallel D_{\text{at}} \parallel^2} \).

The errors of the conventional method and our method versus different \( \beta \) and SNRs are shown in Fig. 1. Here SNR=20 dB and 0 dB cases are considered and \( \beta \) varies from 2 to 4. From the figure, we find that TS-SSD method shows more robustness when \( \beta \) and SNRs change because it obtains \( \hat{D}_{\text{at}} \) using two stages differences technique and TLS solution. Secondly, we consider three methods for fixing \( D_{\text{at}} \), namely, Singular Value Thresholding (SVT) [12], exact completion solver (ECS) [11], and our proposed method. The recovered errors were reported in Fig. 2. Among them, ECS is a closed form solution, which is derived in noiseless case. And it is very sensitive to noise level. SVT is an iterative solution for handling the case with noised \( D_{\text{at}} \). And the performance of
SVT is constrained by several parameters, including step size, number of iterations to achieve convergence, etc. So it has heavy computational burden with low accuracy. The results show that the performance of our proposed method is superior with respect to the other techniques in all listed SNR cases.

Thirdly, the cumulative distribution functions (CDF) of different localization methods, including our proposed SDP method, classical MDS [8], and dwMDS [9], are shown in Fig. 3. We consider that these methods in two cases: $D$ is full recovered and it is not partial recovered. All results are obtained by averaging over 200 independent trials with SNR $= 0$ dB. It is worth observing that our method obtains better performance in the same noise level and it shows more robust than others as the value of SNR decreases. Therefore, our proposed method has good performance in accuracy and robustness.

All simulations were run on a Window 7 desktop computer with a 2.66 GHz Intel Core-i5 Quadcore CPU and 4 GB of RAM. The average run time of these methods are listed in Table I. We find that the complexity of our method is close to that of dwMDS and both of them are higher than classical MDS.

### III. Conclusion

We address a TS-SSD localization algorithm using SDP relaxation. Based on the conventional path loss model, we derive a new linear model using twice difference techniques. A TLS solution and a modified interior point method are used to estimate the unknown distance matrix. Based on the estimated distance matrix, a localization approach using SDP relaxation is derived to obtain the position estimation of target nodes. The proposed method outperforms some existing methods in robustness, model error, and noise level. The simulation results verify the superiority of our proposed method.

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