

① There are two systems, namely A and B, that have the fixed numbers of particles (N_A and N_B , respectively) and are confined in the fixed volume (V_A and V_B , respectively). The energy states of the two systems are $\{E_{1A}, E_{2A}, \dots, E_{iA}, \dots\}$ and $\{E_{1B}, E_{2B}, \dots, E_{iB}, \dots\}$, respectively. Now, the two systems are in direct contact and can exchange heat, and both systems are immersed in a large thermal bath at temperature T . Please derive the expression of chance (P_{ij}) to find A in state E_{iA} and B in state E_{jB} at the same time as a function of T and $E_{1A}, E_{2A}, \dots, E_{iA}, \dots, E_{1B}, E_{2B}, \dots, E_{iB}, \dots$, etc.

② For a grand canonical ensemble containing two types of particles whose chemical potentials are μ_A and μ_B , respectively. Please derive the expression of chance $P_j(N_1, N_2)$ to find the system in a state $E_j(N_1, N_2)$ with N_1 A particles and N_2 B particles and being confined in a fixed volume V and at temperature T .

③ Show that in a two-component, open, isothermal system,

$$\overline{N_1 N_2} - \overline{N_1} \cdot \overline{N_2} = k_B T \left(\frac{\partial \overline{N_1}}{\partial \mu_2} \right)_{V, T, \mu_1} = k_B T \left(\frac{\partial \overline{N_2}}{\partial \mu_1} \right)_{V, T, \mu_2}$$