AN EFFICIENT BLIND TIMING SKEWS ESTIMATION FOR TIME-INTERLEAVED ANALOG-TO-DIGITAL CONVERTERS

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ABSTRACT

In this paper, we propose an efficient blind timing skews estimation (TSE) method for TIADC systems by evaluating the autocorrelation of the analog input signal and the mean squared difference between the output samples of the two adjacent sub-ADCs. By approximating the input signal autocorrelation function using the second order Maclaurin series, an analytical solution of the blind TSE using noise free sub-ADC output model and additive white Gaussian noise model is derived. Experimental results show that the proposed blind TSE method has low computational complexity, good estimation accuracy and robustness to additive Gaussian noise for the analog signals with input frequency lower than half of the Nyquist frequency. The experiment results with the real TIADC output data further validate its good performance.

Index Terms-Time-interleaved ADC, Channel mismatch (CM), bind timing skew estimation (TSE), Autocorrelation

1. INTRODUCTION

High speed and high resolution analog-to-digital converter (ADC) is a key component for many modern electronic systems. For a given fabrication technology and a fixed ADC word length, there is a limit to the maximum achievable sampling frequency (f_s in Hz). Time-interleaved ADC (TIADC) system is an advanced technology allowing achieving higher f_s than that achievable by a single ADC [1]. An M-channel TIADC system is illustrated in Fig. 1. Ideally, M parallel channels are assumed to be linear and identical, which means each sub-ADC should have the same gain, the same sampling interval $T_s = 1/f_s$, and operating at the precise sampling time instants. However, due to the practical implementation constraints, TIADC exhibits the following problems: each channel has slightly different gain leading to "channel gain mismatch", a different bandwidth leading to "channel bandwidth mismatch", a different d.c. offset leading to "channel d.c. offset mismatch", different clock generating circuits and transmission path leading to "timing skews". It is well known that the above channel mismatches (CM) severely reduce the spurious-free dynamic range (SFDR) of the TIADC [2]. In order to compensate these CMs, several effective CM compensation algorithms are proposed [3-6].

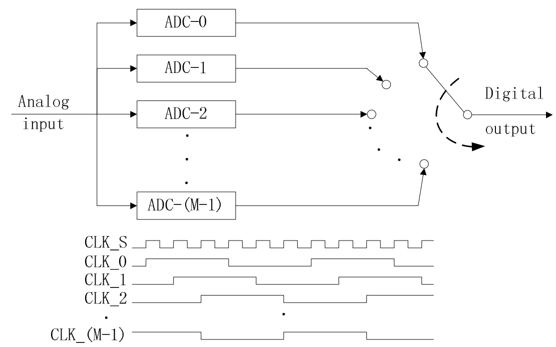

Fig. 1 A time interleaved ADC employing M sub-ADCs

Carefully evaluation reveals that these CM compensation methods are commonly developed with the assumption of knowing the CM parameters [5], which asks for a good measurement of the CM parameters. However, the measurement of the CM parameters using either online or offline approaches is a very challenging task. [3, 7-8]. In principle, the offline approaches need the known input calibration signal. On the contrary, blind estimation methods only request some statistical characters of the input signals in an online manner, which have the advantages of being able to track the CM varying. Literature studies showed that there are several groups working on the blind TSE methods from different perspectives [10-13]. Jamal [10] and Huang [12] employed an oversampling technique and used the alias free bandwidth to estimate the CM parameters in a blind estimation manner. Unfortunately, these methods presented poor scalability. In [11], J. Elbornsson etc. integrated the CM parameter estimation and compensation into a uniform framework known as blind adaptive equalization. The mismatches are compensated using their proposed iterative stochastic gradient minimization algorithm. Study shows that this algorithm requests a high computational load. In conclusion, it seems that, compared to the offline CM parameter measurement methods, the blind estimation methods are less accurate and have much higher computational complexity [5].

In this paper, we carefully evaluate the TIADC working principle, the statistical properties of the analog signal, the autocorrelation of the analog input signal, the mean squared difference between the output samples of the two adjacent sub-ADCs and aim at developing a computation efficient blind TSE algorithm for online TIADC mismatch compensation systems. The analytical solution of the blind TSE using noise free sub-ADC output model and additive white Gaussian noise sub-ADC output model is derived.

2. TIMING SKEW ESTIMATION

2.1. TIADC signal model

(1) Noise free sub-ADC output model (NF-SOM):

A TIADC system shown in Fig. 1 is considered. If we do take the background noise (mainly is quantization noise) into account, the output of the *m*th channel at time index *n*, can be modeled as

$$x_m[n] = g_m s\big((nM+m)T_s + \Delta t_m T_s\big) + O_m \qquad (1)$$

where $s(t)$ is the analog input, T_s is the overall system sampling period, M is the number of sub-ADCs, and $m = 0, \ldots, M-1$. g_m, O_m and Δt_m represents the gain mismatch, amplitude offset and timing skews at *m*th sub-ADC, respectively. The TIADC output is then composed by $x_m[n]$ at the end of the TIADC system, which can be expressed as $y[n]{=}x_{(n \bmod M)}\left[\lfloor \frac{n}{M} \rfloor\right]$, where $\lfloor \cdot \rfloor$ denotes the integer operator. Without loss the generality, in this paper, we focus on the estimation of the timing skews and assume $g_m{=}1$ and $O_m h{=}0h$. With these assumptions, eqn. (1) can be reformulated as

$$x_m[n] = s\big((nM+m)T_s + \Delta t_m T_s\big) \qquad (2)$$

(2) Additive white Gaussian noise Sub-ADC output model (AWGN-SOM):

Working with real TIADC systems, noises cannot be ignored. In the research of TIADC technology, it is a common practice to have following assumption: 1) the noise in each sub-ADC is zero mean additive white Gaussian noise denoted as $e_m[n] \in N(0, \sigma^2)$; 2) noise is independent of input signal; 3) noises from different sub-channels are mutually uncorrelated. As a result, the sub-ADC output signal can be modeled as

$$x_m[n] = s((nM+m)T_s + \Delta t_m T_s) + e_m[n] \qquad (3)$$

2.2. Analytical solution of blind TSE using NF-SOM

Practically, the analog input signal will pass through a lowpass anti-aliasing filter to satisfy the requirement of the Nyquist sampling theory. Therefore, it is reasonable for us to assume that the analog input signal is band limited and quasi-stationary. The mean squared difference (MSD) between the output samples of the two adjacent sub-ADCs at the *n*th time index can be defined as:

$$C_{m,m-1} \equiv E\big\{(x_m[n] - x_{m-1}[n])^2\big\} \qquad (4)$$

where $E\{\cdot\}$ is the expectation operator. By defining the lag as $\tau_m = (1 + \Delta t_m - \Delta t_{m-1})T_s$ and the autocorrelation of $s(t)$ as $R_{ss}(\tau)$, using eqn. (2), eqn. (4) can be expressed as:

$$\begin{aligned} &C_{m,m-1} \\ &= E\big\{x_m^2[n] + x_{m-1}^2[n] - 2x_m[n]x_{m-1}[n]\big\} \\ &= 2\left\{R_{ss}(0) - R_{ss}(\tau_m)\right\} \quad (m = 1, \ldots, M-1) \end{aligned} \qquad (5)$$

It is easy to understand that the rolling property of $R_{ss}(\tau)$ around τ=0 is relevant to the highest component in $s(t)$. It is noted that $s(t)$ is assumed to be band-limited and τ_m is a very small quantity. Using the second order Maclaurin series expansion at τ =0, $R_{ss}(\tau_m)$ in (5) can be approximated by:

$$\begin{aligned} R_{ss}(\tau_m) &\approx \hat{R}_{ss}(\tau_m) \\ &\equiv R_{ss}(0) + R'_{ss}(0)\tau_m + \frac{1}{2}R''_{ss}(0)\tau_m^2 \end{aligned} \qquad (6)$$

It is noted that $R_{ss}(\tau)$ is an even function for a real signal and it has a maximum at τ=0. Besides, we have $R'_{ss}(0) = 0$ and $R''_{ss}(0) < 0$. Substituting (6) into (5), we arrive at

$$C_{m,m-1} \approx -R''_{ss}(0)\tau_m^2 \quad (m = 1, \ldots, M-1) \qquad (7)$$

Obviously, for m=0, eqn. (7) should be expressed as

$$C_{m,m-1} \equiv C_{0,M-1} \approx -R''_{ss}(0)\tau_m^2 \quad (m = 0) \qquad (8)$$

Without loss of generality, we choose sub-channel 0 as the reference channel, thus we have Δt_0=0. Apparently, taking the square root of both sides of (7) and solving these linear equations, we get the analytical approximation of TSE at *m*th sub-channel:

$$\Delta t_m = \frac{\mathbf{A}}{\mathbf{B}} - m \quad (m = 1, \ldots, M-1) \qquad (9)$$

where $\begin{cases} \mathbf{A} = \sum_{i=1}^{m} \sqrt{C_{i,i-1}} \\ \mathbf{B} \equiv \sqrt{-R''_{ss}(0)}T_s = \frac{1}{M}\sum_{i=0}^{M-1} \sqrt{C_{i,i-1}} \end{cases}$

As description above, to estimate the timing skew Δt_m in (9), $C_{m,m-1}$ has to be estimated. Since $C_{m,m-1}$ is independent of *s*(*t*), so we call the method in (9) as a blind TSE method. Study shows that the commonly used estimation technique for $C_{m,m-1}$ is called *N*-point time averaging approach, which is given by

$$\hat{C}_{m,m-1}^N \equiv \begin{cases} \frac{1}{N}\sum_{n=1}^{N}(x_0[n] - x_{M-1}[n-1])^2 & m = 0 \\ \frac{1}{N}\sum_{n=1}^{N}(x_m[n] - x_{m-1}[n])^2 & \text{other} \end{cases} \qquad (10)$$

Using (10), the larger *N* gives higher estimation accuracy of $C_{m,m-1}$. However, larger N will introduce an unacceptable delay for online implementation. Considering the real-time implementation issue and applying the exponentially decaying window approach with forgetting factor as (1-β), we get the recursive estimation as follows

$$\hat{C}_{m,m-1}^N[n] = (1-\beta)\hat{C}_{m,m-1}^N[n-1] + \beta(x_m[n] - x_{m-1}[n])^2 \qquad (11)$$

2.3. Analytical solution of blind TSE using AWGN-SOM

In this section, we will derive the blind TSE method using AWGN-SOM, which is given in (3). Using (3) instead of (2), the mean squared difference (MSD) between $x_m(n)$ and $x_{m-1}(n)$ can be reformulated as

$$C_{m,m-1} \approx -R''_{ss}(0)\tau_m^2 + 2\sigma^2 \qquad (12)$$

In (12), there is an excess term $2\sigma^2$ compared with (7), which reflects the effect introduced by additive noise in $C_{m,m-1}$. Correspondingly, we can derive the analytical solution for TSE at *m*th sub-channel as follows:

$$\Delta t_m = \frac{\widetilde{\mathbf{A}}}{\widetilde{\mathbf{B}}} - m \quad (m = 1, \ldots, M-1) \qquad (13)$$

where $\begin{cases} \widetilde{\mathbf{A}} = \sum_{i=1}^{m} \sqrt{\hat{C}_{i,i-1}^{N} - 2\sigma^2} \\ \widetilde{\mathbf{B}} = \frac{1}{M} \sum_{i=0}^{M-1} \sqrt{\hat{C}_{i,i-1}^{N} - 2\sigma^2} \end{cases}$

To get the TSE using (13), we also need to estimate the noise variance σ^2. In this paper, the eigenvalue decomposition method is used to estimate σ^2. For an M – channel TIADC system, $\mathbf{X}[n]= [x_0[n]\ x_1[n] \ldots x_{M-1}[n]]^T$ can be formed, where $(\cdot)^T$ is the transpose operator. The covariance matrix then can be estimated as

$$\mathbf{R_{XX}} \approx \frac{1}{N} \sum_{n=1}^{N} \mathbf{X}[n]\mathbf{X}[n]^T \tag{14}$$

Conducting eigenvalue decomposition for $\mathbf{R_{XX}}$, we have $\mathbf{R_{XX}} = \mathbf{V\Lambda V}^T$, where the eigenvalues are on the diagonal of the diagonal matrix $\mathbf{\Lambda}$ and the associated eigenvectors form the columns of the matrix $\mathbf{V}$. Since the signal component is much stronger than the additive noise, the noise variance can be estimated by the minimum eigenvalue.

3. SIMULATION AND PERFORMANCE ANALYSIS

3.1. Experiments using simulated data

In this section, we simulated an infinite precision TIADC system which is assumed to have only timing mismatches.

Experiment 1: This experiment is carried out to evaluate the accuracy of the proposed blind TSE method. The input signal is a 32MHz sine wave and we consider NF-SOM case. System parameters are set as follows: f_s=320MHz, N=2048, M=4, β=0.04 and timing skew parameters are [0, -0.06, 0.06, 0.03]. The blind TSE results using (9) and (11) are listed in TABLE I. From TABLE I, it is clear to see that the proposed method can estimate the timing skews effectively with precision up to 96%.

Table I: Blind TSE results

The Proposed method with NF-SOM				
	ADC 0	**ADC 1**	**ADC 2**	**ADC 3**
True Δt_m	**0**	**-0.06**	**0.06**	**0.03**
Estimated Δt_m	0	-0.0577	0.0595	0.0307
Error (%)	0	3.77%	0.833%	2.33%

Experiment 2: This experiment is carried out to evaluate the impact of f_i (input frequency) and N (sample length) on the TSE accuracy using root mean square error (RMSE) measurement, which is defined as

$$RMSE = \left(\frac{1}{M-1} \sum_{m=1}^{M-1} (\Delta t_m - \widehat{\Delta t}_m)^2 \right)^{1/2} \tag{15}$$

where Δt_m and $\widehat{\Delta t}_m$ denote the true and the estimated timing skew for *m*th sub-ADC, respectively. The simulation parameters are set as the same as in Experiment 1. The result is shown in Fig. 2. and Fig. 3, respectively. From Fig. 2, it is noted that the RMSE is inversely proportional to the input frequency. It is happy to see that the RMSE has the upper bound of 0.06Ts when the input frequency approaches the Nyquist frequency (<320MHz/2). From Fig. 3, the RMSE converges when N is larger than 1280.

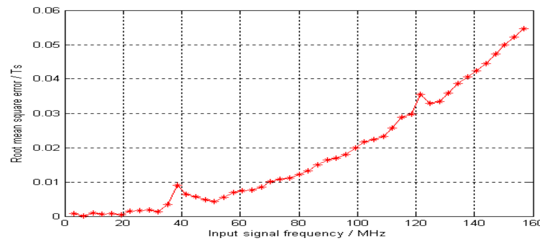

Fig. 2 RMSE vs. input frequency (N=2048)

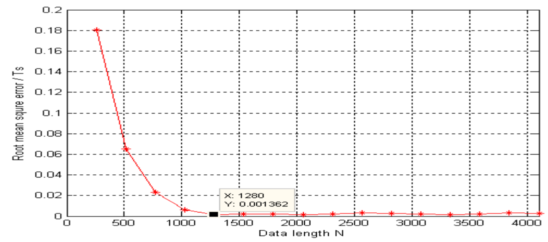

Fig. 3 RMSE vs. N (f_i=32MHz)

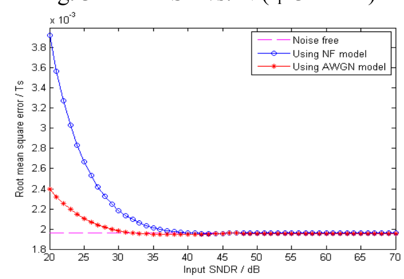

Fig. 4 RMSE performance using NF-SOM and AWGN-SOM under different noise level

Experiment 3: This experiment is conducted to evaluate the performance of the proposed blind TSE method for a TIADC system with additive noise using NF-SOM and AWGN-SOM under different noise level. The signal to noise and distortion ratio (SNDR) is defined as [13]

$$SNDR = 10log_{10}\left(\frac{E\{s^2[k]\}}{E\{d^2[k]\} + E\{e^2[k]\}} \right) \tag{16}$$

where $s[n]$, $d[n]$ and $e[n]$ is the signal, the distortion and noise component, respectively. The parameters are set as the same as in Experiment 1 except β=0.01 and the different noise variance used. The experimental result is plotted in Fig. 4, where the pink curve serves as the reference. This line is RMSE vs. SNDR by the proposed TSE method for the same TIADC system, but with no additive noise. From Fig. 4, it is clear to see the RMSE of the blind TSE method (blue line with circle) using NF-SOM (9) performs worse than that (bold line with star) using AWGN-SOM (13) for all noise level, especially at the SNDR lower than 45dB. This result further illustrates that the time skew estimation method should take the additive noise into account. A small additive noise may result in the time skew estimation bias if we do not consider the noise which does exist in the practical TIADC system.

3.2. Experiments using real TIADC system output data

To further validate the performance of the proposed blind TSE method, we carried out one experiment using the captured output data from a 4×80Msps 12-bit TIADC system developed by our team. Fig. 5 shows our TIADC system prototype. The data was collected with 48.65MHz input sine wave. The timing skews measured using our proposed method is [0, -0.0621, 0.0575, 0.0344]. With these timing skew parameters, the normalized output spectrums of the TIADC system with and without timing mismatch compensation are plotted in Fig. 6, where a multi-rate filter bank based timing mismatch compensation method is used [4]. From Fig. 6 (b), we can clearly see that the spurs in Fig. 6 (a) due to timing skews have been effectively suppressed more than 20dB, which means the system SFDR has been increased 20.8dB. This result further validates the timing skew estimation accuracy of our proposed method.

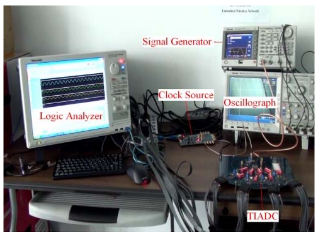

Fig. 5 TIADC System prototype

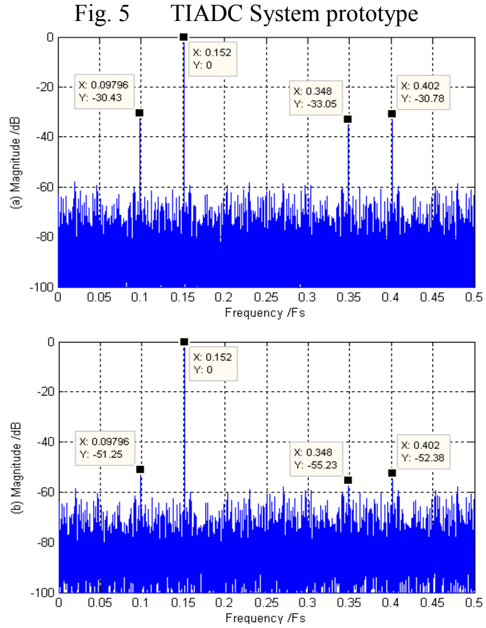

Fig. 6 (a): uncompensated; (b): compensated

4. CONCLUSION

Blind timing skew estimation of TIADC system needs no calibration signal and can be implemented online. In this paper, a blind TSE method was derived systematically for noise free and additive white Gaussian noise conditions. Experimental results with both simulated and captured real TIADC output data validate the efficiency and accuracy of the proposed blind TSE method. It can be concluded that the proposed blind TSE method provides good estimation accuracy with low complexity when the relative input frequency ratio is smaller than half of the Nyquist frequency. Further study is requested to improve the estimation accuracy for higher input frequency.

5. ACKNOWLEDGMENT

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