

Blind Timing Skew Estimation Based on Spectra Sparsity and All Phase FFT for Time-Interleaved ADCs

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Abstract—A time-interleaved analog-to-digital converter (TIADC) is a promising solution for high speed and high resolution ADC. Blind timing skew estimation (TSE) is one of its key techniques for implementing online timing skew mismatch compensation digitally. Assuming the input signal of the TIADC is of spectral sparsity, in this paper, an efficient blind TSE algorithm is developed by employing the all phase Fast Fourier Transform (ApFFT) technique to obtain the accurate phase spectral estimation, which results in the ApFFT-SS-BLTSE algorithm. Experimental results show that, compared to the SS-BLTSE algorithm in [1], the proposed ApFFT-SS-BLTSE algorithm requires less computational cost and is able to offer high TSE accuracy for both narrow and wideband source signals in the presence of additive noise.

Keywords—Timing skews, Time-Interleaved Analog-To-Digital Converters, blind estimation, All Phase Fast Fourier Transform, Undersampling

I. INTRODUCTION

Time-Interleaved Analog-to-Digital Converter (TIADC) structure is one of the most promising techniques to realize high speed and high resolution analog-to-digital converter (ADC) [2]. As shown in Fig.1, a TIADC is formed by using M identical parallel ADCs (sub-ADCs), where the m^{th} channel ADC offers a sampling rate of f_s/M with the sampling clock delayed by mT_s . As a result, an overall sampling rate of f_s can be achieved by the TIADC.

However, due to the practical implementation constraints, TIADC exhibits some channel mismatches, such as channel gain mismatch, channel bandwidth mismatch, channel D.C. offset mismatch, and channel timing skews mismatch [3]. It is well known that the above channel mismatches severely reduce the spurious-free dynamic range (SFDR) of the TIADC. Among these mismatches, the compensation of timing skew mismatch has been subject to active research, but most of related conventional algorithms ask for an accurate estimate of the timing mismatch skew [4]-[7]. An adaptive calibration method is proposed in [8], which is a training method and suitable for high-resolution applications since it is capable of correcting general linear mismatches, but at the cost of system suspension during

each calibration. On the other hand, blind TSE methods are more favorable because they are capable of tracking possibly varying mismatches, while operating in an online manner. However, it is a very challenging task because only the statistical information of the input signal is assumed known. Literature studies show that there are several groups working on the blind TSE methods [1, 9-13], which may have their own limitations. For example, 1) high computation complexity is required [10]; 2) the approach in [11] is only suitable for two channels TIADC; 3) the oversampling of input signals is needed [12]; 4) the energy of the input signals is required to be concentrated on low frequency components [13]. In our previous work [1], the proposed SS-BLTSE algorithm works well and is able to accurately estimate timing skews. Moreover, SS-BLTSE has no requirements on the TIADC system except that the input signal spectrum should be of certain sparsity. Carefully evaluation shows that an N -point FFT is needed in the SS-BLTSE algorithm to determine the nonoverlapping frequency component and its phase information. Moreover, for SS-BLTSE algorithm, a larger N , usually a more accurate TSE will be. As a result, there is a tradeoff between its computational cost and TSE accuracy.

To overcome this limitation, in this paper, we propose a new approach to achieve the phase spectral estimation based on all phase Fast Fourier Transform (ApFFT) technique and accordingly a new algorithm called ApFFT-SS-BLTSE algorithm is derived. The rest part of this paper is structured as follows. Section 2 gives the basics for blindly estimating the timing skews of the TIADC system. In Section 3, the proposed ApFFT-SS-BLTSE algorithm is detailed. The experiments and results are given in Section 4 and the conclusions are drawn in Section 5.

II. THE BLIND TSE ALGORITHM FOR TIADC

For the sake of presentation, we will first summarize the common assumptions and definitions considered in developing the blind TSE algorithm as follows [1]: 1) Input signal $x_d(t)$ is bandlimited with the highest frequency smaller than $f_s/2$; 2) The system parameters of the TIADC system are known, such as the number of sub ADCs (M), the overall sampling frequency (f_s); 3) The discrete output signals ($y_m(n)$, $y(n)$) are available; 4) No

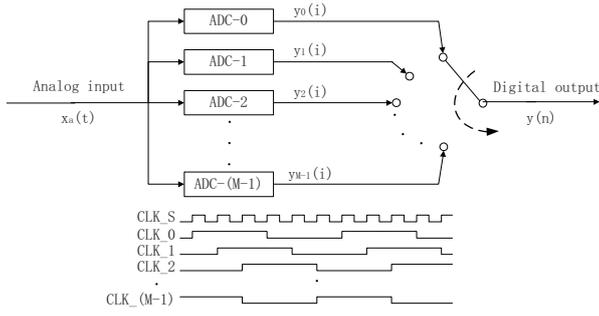


Figure 1. Conceptual illustration of the TIADC with $M=4$.

channel D. C. offset and gain mismatches are considered, since they can be compensated using the simple subtraction and scalar gain approaches; 5) No additive noise is considered; 6) As the timing skew is usually much smaller than the sampling period (smaller than 10%), the input analog signal is much stronger than the disturbance introduced by TIADC channel timing skews.

Literature review shows that the above assumptions are commonly satisfied in the research of the TIADC technology.

A. Data Model of TIADC System

According to the working principle of the TIADC system as shown in Fig. 1, sub-ADCs operate in the undersampling at the factor of M compared to the overall TIADC system. The relation between $x_a(t)$, $y(n)$ and $y_m(n)$ of the TIADC in time and spectrum domain is described as follows [1].

From Fig 1, it is straightforward to get the time domain expression of the output of the m^{th} sub-ADC:

$$y_m(n) = x_a(t) \Big|_{t=nT_{ss} + mT_s + \Delta t_m} \quad (1)$$

where $n=0,1,\dots,N/M-1$, $m=0,1,\dots,M-1$, T_{ss} is the sampling period of the sub-ADCs, N is the total number of samples for TIADC and m is the channel index, Δt_m is the timing skew for m^{th} sub-ADC. The digital frequency ϖ associated with f_{ss} has the following relation with the analog frequency Ω :

$$\varpi = \Omega / f_{ss} = \Omega T_{ss} = 2\pi f / f_{ss} \quad (2)$$

From (1) and (2), the DTFT of $y_m(n)$ can be expressed as

$$Y_m(e^{j\varpi}) = \frac{1}{T_{ss}} \sum_{k=-\infty}^{+\infty} X_a \left(e^{j\left(\frac{\varpi}{T_{ss}} - \frac{2\pi k}{T_{ss}}\right)} \right) e^{j\left(\frac{\varpi}{T_{ss}} - \frac{2\pi k}{T_{ss}}\right)(m+\Delta t_m)T_s} \quad (3)$$

And the DTFT of $y(n)$ is given by [3]:

$$Y(e^{j\omega}) = \frac{1}{T_{ss}} \sum_{k=-\infty}^{+\infty} \sum_{m=0}^{M-1} \left(e^{j\omega \Delta t_m} e^{-j\frac{2\pi k}{M}m} \right) X_a \left(e^{j\left(\frac{\omega}{T_s} - \frac{2\pi k}{T_{ss}}\right)} \right) \quad (4)$$

where the digital frequency ω associated with f_s is given by

$$\omega = \Omega T_s = 2\pi f / f_s \quad (5)$$

B. Blind Estimation of the Timing Skews

In order to explicitly indicate the 2π periodic extension of the fundamental interval of ω and ϖ , here we define

$$\omega_{\langle 2\pi \rangle} = ((\omega + \pi) \bmod 2\pi) - \pi \quad (6)$$

As shown in [1], the closed-form expression of the relative timing skew Δt_m is given as:

$$\Delta t_m = \frac{(P_m(j\omega_p) - P_0(j\omega_p))_{\langle 2\pi \rangle}}{j\left(\frac{\omega_p}{M} + 2(k_p - 1)\frac{\pi}{M}\right)_{\langle 2\pi \rangle}} - m, \omega_p \in (0, 2\pi) \quad (7)$$

where $P_m(j\omega_p)$ is the phase spectrum of $Y_m(e^{j\varpi})$, $m=0,1,\dots,M-1$, and ω_p is the nonoverlapping frequency component, which need to be estimated. From (7), it is clear that the accuracy of the phase spectra estimation determines the estimation accuracy of the timing skews.

In our previous work, we proposed a N -point FFT-based method to estimate ω_p , where (N/M) -point FFT has been used to compute the sub-ADC spectra $Y_m(e^{j\varpi})$ in (3), which resulted in the SS-BLTSE algorithm. It is noted that the SS-BLTSE algorithm works well when the input spectral sparsity is less than $2/M$ and when N is larger than 65536 for testing signals [1]. We also observed that the TSE accuracy decreases with the increase of the input spectrum sparsity and decrease of the data length for FFT. However, preliminary performance analysis suggests that the FFT-based spectral estimation has some limitations, such as ‘‘spectrum leakage’’ due to windowing and Picket fence effect due to the discretization of the spectrum, where the spectral leakage is a continuous function of the frequency and will lead to a spread of spectrum in different frequency bands [14]. These two factors will affect the estimation accuracy of ω_p and phase spectra. Choosing proper window function and increasing the data length are effective approaches to reduce the spectrum leakage, at the expense of high computational cost. This may give the restriction of online applications. Another possible way to reduce the spectrum leakage is to use decaying window functions such as Hamming window in order to achieve a better tradeoff between the computational cost and the performance. According to the analysis above, in this paper, we make the effort to improve the estimation of the phase spectral.

III. THE ApFFT-SS-BLTSE ALGORITHM

It is encouraged to note that a recently proposed ApFFT spectral estimation technique is able to provide better solution to estimate the phase spectra.

A. ApFFT Spectral Estimation Technique

The ApFFT is a new spectral estimation method firstly proposed by Wang in [15]. Compared with traditional FFT-based spectral estimation methods, ApFFT spectral estimation method has less spectrum leakage, high spectrum estimation accuracy and a unique phase invariant property.

First of all, let's define a $(2N-1)$ -length data vector

$$\mathbf{x} = [x(-N+1), x(-N+2), \dots, x(0), \dots, x(N-1)] \quad (8)$$

All N -length shifted data vectors including $x(0)$ is formed:

$$\begin{aligned} \mathbf{z}_0 &= [x(-N+1), x(-N+2), \dots, x(0)] \\ \mathbf{z}_1 &= [x(-N+2), x(-N+3), \dots, x(1)] \\ &\dots \\ \mathbf{z}_{N-1} &= [x(0), x(1), \dots, x(N-1)] \end{aligned} \quad (9)$$

Applying the cyclic shifting operation on N vectors obtained in (9) gives another set of N -length data vectors:

$$\begin{aligned} \mathbf{x}_0 &= [x(0), x(-N+1), \dots, x(-1)] \\ \mathbf{x}_1 &= [x(0), x(1), x(-N+2), \dots, x(-1)] \\ &\dots \\ \mathbf{x}_{N-1} &= [x(0), x(1), x(2), \dots, x(N-1)] \end{aligned} \quad (10)$$

From (10), by averaging $\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{N-1}$, a new N -length data vector can be formed

$$\mathbf{x}_{ap} = \frac{[Nx(0), (N-1)x(1) + x(-N+1), \dots, x(N-1) + (N-1)x(-1)]}{N} \quad (11)$$

The N -length data vector \mathbf{x}_{ap} computed in (11) is called the all phase data vector of a $(2N-1)$ -length data vector \mathbf{x} . The FFT of the data vector \mathbf{x}_{ap} gives the all phase spectral estimation of a $(2N-1)$ -length data vector \mathbf{x} , which is named as the ApFFT spectral estimation method. For example, as shown in (8), a N -point data vector \mathbf{x} can be formed from a single frequency complex exponential sequence $x(n) = e^{j\omega_0 n + \phi}$, where ω_0 and ϕ are the digital frequency and phase, respectively. The closed-form N -point FFT of \mathbf{x} is

$$\begin{aligned} X(k) &= \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi} e^{j2\pi k_0 n/N} e^{-j2\pi kn/N} \\ &= \frac{1}{N} e^{j\phi} \sum_{n=0}^{N-1} e^{j2\pi(k_0-k)n/N} = \frac{1}{N} e^{j\phi} \frac{1 - e^{j2\pi(k_0-k)}}{1 - e^{j2\pi(k_0-k)/N}} \quad (12) \\ &= \frac{1}{N} \frac{\sin[\pi(k_0-k)]}{\sin[\pi(k_0-k)/N]} e^{j[\phi + (1-1/N)\pi(k_0-k)]} \end{aligned}$$

where the signal frequency index $k_0 = \text{floor}(\omega_0 N / (2\pi))$ and $k=1, \dots, N$. With the shift property of FFT, the FFT spectrum of \mathbf{z}_i and \mathbf{x}_i ($i=0, \dots, N-1$) has the relation of $X_i(k) = Z_i(k) e^{j2\pi ki/N}$ ($k=0, \dots, N-1$), therefore, the FFT spectrum of \mathbf{x}_{ap} can be derived as

$$\begin{aligned} X_{ap}(k) &= \frac{1}{N} \sum_{i=0}^{N-1} Z_i(k) e^{j2\pi ki/N} \\ &= \frac{1}{N^2} \sum_{i=0}^{N-1} \left(\sum_{n=0}^{N-1} e^{j\phi} e^{j2\pi k_0(n-i)/N} e^{-j2\pi kn/N} \right) e^{j2\pi ki/N} \\ &= \frac{1}{N^2} e^{j\phi} \sum_{i=0}^{N-1} e^{-j2\pi(k_0-k)i/N} \sum_{n=0}^{N-1} e^{j2\pi(k_0-k)n/N} \\ &= \frac{1}{N^2} \frac{\sin^2[\pi(k_0-k)]}{\sin^2[\pi(k_0-k)/N]} e^{j\phi} \end{aligned} \quad (13)$$

By comparing (13) with (12), we can see clearly that the phase of $X_{ap}(k)$ computed by the ApFFT method is exactly equal to that of the input data. Meanwhile the phase of $X(k)$ computed by the FFT method has extra terms, which can be viewed as phase estimation error. While the spectrum amplitude, $|X_{ap}(k)|$ is equal to the square of $|X(k)|$, which means the attenuation of the sidelobes in $|X_{ap}(k)|$ will be greater than those in $|X(k)|$. Hence, we can conclude that the spectrum leakage of ApFFT spectral estimation method will be less than that of the FFT spectrum estimation method.

B. Proposed ApFFT-SS-BLTSE Algorithm

In this section, a new blind timing skews estimation algorithm named as ApFFT-SS-BLTSE will be developed in details in this subsection.

PART 1: All phase spectrum estimation

In this part, as shown in Fig.1, we will use the ApFFT spectral estimation method to estimate the spectra of $y(n)$ and $y_m(n)$, respectively. The procedure is listed as follows:

- 1) Using (8) and $(2N-1)$ -length output data of the TIADC to form the N -length data vector \mathbf{x} ;
- 2) Using (9) to form the shifted data vector \mathbf{z}_i ($i=0, \dots, N-1$);
- 3) Using (10) to form the cyclic shifted data vector \mathbf{x}_i ;
- 4) Using (11) to form the all phase data vector \mathbf{x}_{ap} ;
- 5) Perform N -point FFT of \mathbf{x}_{ap} to obtain the primary all phase spectrum estimation $Y'(e^{j\omega})$;
- 6) Get the estimated spectrum $Y(e^{j\omega})$ by using preprocess:

$$Y(e^{j\omega}) = \begin{cases} 0 & |Y'(e^{j\omega})| \leq \varepsilon_x \max(|Y'(e^{j\omega})|) \\ Y'(e^{j\omega}) & \text{others} \end{cases} \quad (14)$$

where the threshold parameter ε_x is set to be 0.06 [1].

- 7) Similarly, we can estimate all phase spectra $Y_m(e^{j\omega})$ of $y_m(n)$ ($m=0, 1, \dots, M-1$) using the same procedure for estimating $Y(e^{j\omega})$ except that we need to use $(2N/M-1)$ -length output data of the sub-ADC ($y_m(n)$) to form the N/M -length all phase data vector \mathbf{x}_{ap} .

PART II: The determination of nonoverlapping frequency component ω_p

In this part, we will use the method proposed in [1] to determine the nonoverlapping frequency component ω_j :

1) For a digital frequency ω_i in the range of $(0\sim 2\pi)$, the amplitude frequency spectrum (AFS) can be calculated by

$$N_x(k, i) = \left| Y \left[\exp \left(j \left(\omega_i / M + 2\pi(k-1) / M \right) \right) \right] \right| \quad (15)$$

where $k = 1, \dots, N$, and $Y(e^{j\omega})$ is estimated in (14).

2) To find the nonzero value of $N_x(k, i)$ for ω_i using the data obtained in (15). If there is only one $N_x(k_p, i) \neq 0$, then we define the j^{th} nonoverlapping frequency as

$$N_p(k_p, j) = N_x(k_p, i), \quad \omega_p(j) = \omega_i \quad (16)$$

Repeat the same procedure for each ω_i and then we obtain the nonoverlapping frequency set as $\{\omega_p(j), j=1, \dots, H_L, H_L \leq N/M\}$ and its corresponding AFS set $\{N_p(k_p, j), j=1, \dots, H_L\}$.

3) Find the maximum of the AFS set $\{N_p(k_p, j), j=1, \dots, H_L\}$, correspondingly, the nonoverlapping frequency ω_p and its index k_p can be determined.

As the result, the proposed ApFFT-SS-BLTSE algorithm is summarized as follows:

- 1) Calculate the all phase spectrum $Y(e^{j\omega})$ and the all phase spectrum $Y_m(e^{j\omega})$ using the method described in PART I;
- 2) Find the nonoverlapping frequency component ω_p and its index k_p as shown in PART II, meanwhile compute the phase spectra of $Y(e^{j\omega_p})$ and $Y_m(e^{j\omega_p})$, respectively;
- 3) Compute the relative timing skews Δt_m by (7).

IV. EXPERIMENT RESULTS

To evaluate the performance of the proposed ApFFT-SS-BLTSE algorithm, several simulations have been conducted. A 4-channel TIADC system ($M=4$) is used and only timing mismatch is considered, where $f_s=4\text{GHz}$ and relative timing skews are $\Delta t_0=0$, $\Delta t_1=0.03f_s$, $\Delta t_2=0.05f_s$ and $\Delta t_3=-0.02f_s$.

Experiment 1: This experiment is carried out to compare the TSE capability of the ApFFT-SS-BLTSE algorithm with that of the SS-BLTSE algorithm proposed in [1] under noiseless condition. To reflect the input spectral sparsity, a more general multi-frequency signal is used, which is denoted by:

$$x_a(t) = \sum_{k=1}^{32} a_k \sin(2\pi f_k t + \theta_k) \quad (17)$$

where f_k , a_k and θ_k are uniform distributed random sequences, f_k ranges from 1Hz to 1.95GHz, a_k ranges from 5 to 10 and θ_k ranges from $-\pi$ to π , respectively; no additive noises are considered. The simulation results are shown in Table I. From Table I, it can be seen that the performance of the ApFFT-SS-BLTSE algorithm significantly outperforms the SS-BLTSE algorithm. Besides, the computational cost of the ApFFT-SS-BLTSE algorithm is about half of that of the SS-BLTSE algorithm. As an example, if $N=65536$, the cost of the ApFFT-SS-BLTSE algorithm will be about one 32768-point FFT and

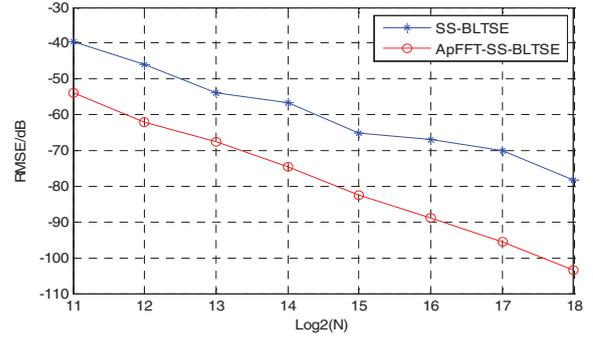


Figure 2. RMSE vs N

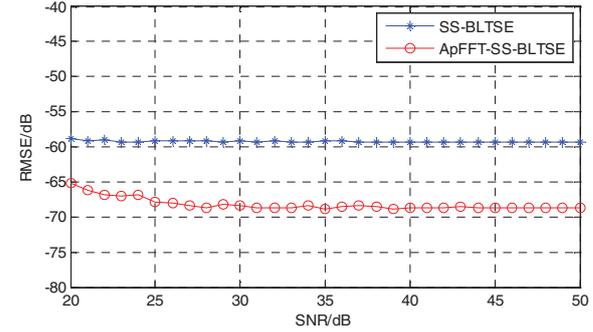


Figure 3. RMSE vs SNR

TABLE I. THE TSE RESULTS ($N=65536$, $\epsilon_r=0.06$, HAMMING WINDOW)

| Channel index m | | 0 | 1 | 2 | 3 |
|-------------------|------------------------|---|--------|--------|---------|
| True Δt_m | | 0 | 0.03 | 0.05 | -0.02 |
| SS-BLTSE | Estimated Δt_m | 0 | 0.0305 | 0.0502 | -0.0196 |
| | Error (%) | 0 | 1.67% | 0.40% | 2.00% |
| ApFFT-SS-BLTSE | Estimated Δt_m | 0 | 0.0300 | 0.0500 | -0.0199 |
| | Error (%) | 0 | 0.00% | 0.00% | 0.50% |

four 8096-point FFT, but the cost of the SS-BLTSE algorithm will be one 65536-point FFT and four 16384-points FFT. Hence, the proposed ApFFT-SS-BLTSE algorithm is more efficient and less sensitive to the signal spectrum sparsity than that of the SS-BLTSE algorithm, which are favorable properties for real applications.

Experiment 2: This experiment is carried out to evaluate the influence of N on the TSE performance of the ApFFT-SS-BLTSE and the SS-BLTSE algorithms. The following root mean square error (RMSE) is used to assess the performance of the two algorithms

$$RMSE = 20 \log_{10} \sqrt{\frac{1}{M-1} \sum_{m=1}^{M-1} (\Delta t_m - \Delta t'_m)^2} \quad (\text{dB}) \quad (18)$$

where Δt_m and $\Delta t'_m$ denote the true and the estimated timing skews for m^{th} sub-ADC, respectively. The simulation setups are the same as those used in Experiment 1 except N varies from 2^{11} to 2^{18} (that is 2048 to 262144). The simulation results are shown in Fig.2. It is clear to see that the ApFFT-SS-BLTSE algorithm outperforms the SS-BLTSE algorithm in all settings, and the

ApFFT-SS-BLTSE algorithm performs well when the data-length used is larger than 2^{12} .

Experiment 3: This experiment is carried out to evaluate the influence of the additive noise on the TSE accuracy of the SS-BLTSE and ApFFT-SS-BLTSE algorithms. The same simulation parameters as those in Experiment 1 are used. The simulation results obtained by 100 independent runs for each SNR are shown in Fig.3. From Fig. 3, we can see that two algorithms are not sensitive to additive noise under this experiment setup. Obviously, the ApFFT-SS-BLTSE algorithm is superior to SS-BLTSE algorithm in terms of estimation accuracy and computational cost for all SNRs.

V. CONCLUSION

Motivated by the accurate and efficient phase spectrum estimation of the recently proposed all phase FFT (ApFFT) spectrum estimation technique, a new blind timing skew estimation algorithm, named as ApFFT-SS-BLTSE algorithm, is developed where the phase spectra for estimating the timing skews is determined by ApFFT in an efficient way. Simulations show that the ApFFT-SS-BLTSE algorithm is able to offer the good TSE accuracy under different noise conditions. Moreover, compared to the SS-BLTSE algorithm [1], ApFFT-SS-BLTSE holds the merits of low computation cost, robustness to the additive noise and higher TSE accuracy. It is confident to conclude that our proposed ApFFT-SS-BLTSE algorithm is an effective solution for blindly estimating the timing skews of TIADCs.

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