Two Stages Signal Strength Difference Localization Algorithm using SDP Relaxation

Xiansheng Guo, Lei Chu, Yiming Pi
Dept. of Electronic Engineering
University of Electronic Science Technology of China
Chengdu, China
xsguo@uestc.edu.cn

Yuexian Zou
School of Electronic and Computer Engineering
Peking University
Shenzhen, China
zouyx@pkusz.edu.cn

***Abstract*—We present a novel two stages signal strength difference (TS-SSD) localization algorithm in this letter. A new model using TS-SSD technique is derived to eliminate the effects of path loss exponent and unknown transmit power. And a total least squares (TLS) solution is given to estimate the distances between anchor and target nodes. Then a low-rank matrix completion framework is established to estimate the distances among all the unknown target nodes. After obtaining all distance measurements, a simple but robust approach based on Semidefinite Programming (SDP) relaxation is applied to estimate the location of target nodes. Compared with some existing algorithms, the proposed approach has three-fold merits: firstly, it locates multiple targets using less anchor nodes simultaneously; secondly, it shows more robustness to path model error and noise level; thirdly, it has higher localization accuracy than some existing methods. Simulation results illustrate the superiority of our proposed algorithm.**

***Index Terms*—Localization; two stages signal strength difference (TS-SSD); matrix completion; SDP;**

I. Introduction

Localization is one of the essential modules of many mobile wireless applications [1, 2]. Because Received Signal Strength (RSS) measurement can be easily obtained on most off-the-shelf equipments, such as WiFi- or ZigBee compatible devices, a majority of previous localization approaches with RSS as a metric for location determination are proposed in the past two decades.

The accuracy of some existing RSS-based localization algorithms depends highly on the knowledge of relevant parameters, including transmit power, path loss exponent, and noise level [1–3]. Unfortunately, securing these parameters accurately in unknown environment is time intensive. As a consequence, two kinds of methods are presented to overcome the drawback. One tries to estimate the unknown node locations directly without knowing the path loss exponent [4, 5]. The performance of these methods is good in high SNR case, but they cannot estimate multiple targets simultaneously. The other is fingerprinting based method which does not need to estimate the nuisance parameters [6]. It can improve the accuracy of localization to some extent, but the fingerprinting database building process is a time consuming job. Signal strength difference (SSD) technique is a good way to mitigate the effect of unknown transmit power [7] and it improves the performance of localization greatly but it still belongs to fingerprinting based localization framework.

Instead of estimating the target locations from the noisy RSS measurements directly, we recover the distance matrix among all nodes from noisy RSS measurements firstly, and the locations of target nodes are estimate from the recovered distance matrix using SDP framework. The main steps of our proposed algorithm are as follows:

a) derive a new model from the conventional path loss model using two stages SSD approach;
b) a TLS solution is used to calculate the distances between anchor nodes and target nodes;
c) estimate the distances among all target nodes using low-rank matrix completion method;
d) target locations are estimated from the distance matrix among all nodes using SDP relaxation.

Compared with some existing methods, such as multidimensional scaling (MDS) method [8], and distributed weighted-multidimensional scaling (dwMDS) method [9]. the merits of our method are threefold: firstly, it locates multiple targets using less anchor nodes simultaneously; secondly, it shows more robustness to path model error and noise level; thirdly, it has higher localization accuracy than some existing methods.

A. The proposed algorithm

B. RSS measurements model and distance matrix

Given M ($M \geq 3$) anchor nodes (We suppose that at least three of M can communicate with each other) in a wireless localization system with locations $\boldsymbol{O} = [\boldsymbol{o}_1, \boldsymbol{o}_2, \cdots, \boldsymbol{o}_M]$, and N target nodes whose locations $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N]$ are to be determined.

Let $\underline{d}_{i,j} = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2$ denote the distance between target nodes i and j, $d_{k,j} = \|\boldsymbol{o}_k - \boldsymbol{x}_j\|_2$ denote the distance between anchor node k and target node j, and $\overline{d}_{k,l} = \|\boldsymbol{o}_k - \boldsymbol{o}_l\|_2$ denote the distance between anchor nodes k and l, where $\|\cdot\|_2$ denotes the ℓ_2-norm. The nonnegative and symmetric distance matrix among all nodes with zero diagonal can be expressed as

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_{\text{aa}} & \boldsymbol{D}_{\text{at}} \\ \boldsymbol{D}_{\text{ta}} & \boldsymbol{D}_{\text{tt}} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{D}_{\text{aa}} = (\overline{d}_{i,j}^2)$, $\boldsymbol{D}_{\text{at}} = \boldsymbol{D}_{\text{ta}}^T = (d_{j,k}^2)$, and $\boldsymbol{D}_{\text{tt}} = (\underline{d}_{k,l}^2)$.

Suppose that $P_{i,j}$ and P_0 are the RSS measurements (measured at anchor node i) at an arbitrary distance $d_{i,j}$ and a close-in reference distance d_0 from the node j, respectively. From the log-normal shadowing model [1], we have

$$P_{i,j} = P_0 - 10\beta \log_{10} \frac{d_{i,j}}{d_0} + n_i, \tag{2}$$

where β denotes the path loss exponent, n_i is empirically modeled as a lognormal random variable with zero mean and variance σ_i^2. The problem of localization using RSS measurements is to estimate the locations of target nodes $\boldsymbol{X}$ from $P_{i,j}$. Obviously, if we can estimate $\boldsymbol{D}$ from the noisy RSS measurements, the location of target nodes can be estimated by SDP technique.

C. The estimates of $\boldsymbol{D}_{at}$ and $\boldsymbol{D}_{ta}$ using two stages SSD (TS-SSD) method

If we neglect the noise elements in (2), we have

$$\hat{P}_{i,j} = P_0 - 10\beta \log_{10} \frac{d_{i,j}}{d_0}. \tag{3}$$

Let $\overline{P}_{m,i}$ be the RSS between the mth and ith ($i = 1, 2, \cdots, M, i \neq m$) anchor nodes, and $P_{m,j}$ be the RSS between the mth anchor and the jth ($j = 1, 2, \cdots, N$) target node. Take the mth anchor node as the reference node, we compute the signal strength difference by subtracting $P_{m,j}$ from $\overline{P}_{m,i}$, yields

$$\overline{P}_{m,i} - P_{m,j} = -10\beta \log_{10}^{\frac{\overline{d}_{m,i}}{d_{m,j}}}. \tag{4}$$

Usually, β can be assumed a constant in a short period of observation time [4]. Hence, we can compute the signal strength difference for the second time,

$$\frac{\overline{P}_{m,m+1} - P_{m,j}}{\overline{P}_{m,k} - P_{m,j}} = \frac{\log_{10}\overline{d}_{m,m+1} - \log_{10}d_{m,j}}{\log_{10}\overline{d}_{m,k} - \log_{10}d_{m,j}}, \tag{5}$$

where $k = 1, 2, \cdots, M$ ($k \neq m, k \neq m+1$) is a subset of i. If $m = M$, we set $m + 1 = 1$. Let $\overline{z}_{m,i} = \log_{10}^{\overline{d}_{m,i}}$ and $z_{m,j} = \log_{10}^{d_{m,j}}$, respectively, (5) can be rewritten as

$$\begin{aligned}\left(\overline{P}_{m,m+1} - \overline{P}_{m,k}\right) z_{m,j} = \left(\overline{P}_{m,m+1} - P_{m,j}\right) \overline{z}_{m,k} \\ - \left(\overline{P}_{m,k} - P_{m,j}\right) \overline{z}_{m,m+1}.\end{aligned} \tag{6}$$

Denoting the $N \times 1$ distance vector $\boldsymbol{z}_m = [z_{m,1}, z_{m,2}, \cdots, z_{m,N}]^T$, (6) can be reformulated as

$$\boldsymbol{A}_m \boldsymbol{z}_m = \boldsymbol{b}_m, \tag{7}$$

where $\boldsymbol{A}_m \in \mathcal{R}^{Q\times N}$ ($Q = (M-2) \times N$) is given by $\boldsymbol{A}_m = \text{diag}(\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_N)$ and all the $\boldsymbol{a}_j = \mathbf{1}\overline{P}_{m,m+1} - \overline{\boldsymbol{p}}_m$ are the same vector with dimension $(M-2) \times 1$. The $Q \times 1$ measurement vector $\boldsymbol{b}_m = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_N]^T$ and its jth element $\boldsymbol{b}_j = \left(\overline{P}_{m,m+1} - P_{m,j}\right) \overline{\boldsymbol{z}}_m - (\overline{\boldsymbol{p}}_m - \mathbf{1}P_{m,j}) \overline{z}_{m,m+1}$, where $\mathbf{1}$ is an $(M-2) \times 1$ all ones vector , $\overline{\boldsymbol{z}}_m = [\overline{z}_{m,1}, \cdots, \overline{z}_{m,k}, \cdots, \overline{z}_{m,M}]^T, k \neq m, m+1$, and $\overline{\boldsymbol{p}}_m = \left[\overline{P}_{m,1}, \cdots, \overline{P}_{m,k}, \cdots, \overline{P}_{m,M}\right]^T, k \neq m, m+1$. Observing $\boldsymbol{A}_m$ is an over-determined matrix, (7) has a least squares (LS) solution. However, the new model is derived under noise free environment, errors exist on both sides of (7). To obtain a more robust solution, we consider

$$(\boldsymbol{A}_m + \boldsymbol{E}_m) \boldsymbol{z}_m = \boldsymbol{b}_m + \boldsymbol{e}_m, \tag{8}$$

where $\boldsymbol{E}_m$ and $\boldsymbol{e}_m$ are error matrix and error vector of $\boldsymbol{A}_m$ and $\boldsymbol{b}_m$, respectively. Obviously, (8) can be rewritten as

$$(\boldsymbol{F}_m + \boldsymbol{G}_m) \boldsymbol{d}_m = \mathbf{0}, \tag{9}$$

where $\boldsymbol{F}_m = [-\boldsymbol{b}_m, \boldsymbol{A}_m]$ is an extended matrix, $\boldsymbol{G}_m = [-\boldsymbol{e}_m, \boldsymbol{E}_m]$ is a disturbance matrix, and $\boldsymbol{d}_m = [1, \boldsymbol{z}_m]^T$ is a $(N+1) \times 1$ vector to be determined.

Let singular value decomposition (SVD) of $\boldsymbol{F}_m$ be $\boldsymbol{F}_m = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$, with $\boldsymbol{U}$ and $\boldsymbol{V}$ be the left and right singular vectors, respectively. And $\boldsymbol{\Sigma}$ is a $Q \times (N+1)$ matrix whose elements are zero except possibly along its main diagonal. These nonnegative diagonal elements are ordered such that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{N+1}$. And the right singular vector which corresponds to these singular values is $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_{N+1}]$ and its $(N+1)$th element can be expressed as $\boldsymbol{v}_{N+1} = [v(1, N+1), v(2, N+1), \cdots, v(M, N+1)]^T$. The TLS solution of (9) can be given by the singular vector which corresponds to the smallest singular value σ_{N+1}

$$\hat{\boldsymbol{z}}_m^{\text{TLS}} = \frac{1}{v(1, N+1)} [v(2, N+1)]. \tag{10}$$

Note that if the number of the smallest singular value is more than one, the approximate TLS solution can be found using some modified smoothing techniques [10], which shows more robustness than the conventional LS solution.

Once $\hat{\boldsymbol{z}}_m$ is obtained, the distances can be given by $\hat{d}_m = 10^{\hat{\boldsymbol{z}}_m^{\text{TLS}}}$ and we can build a distance matrix $\boldsymbol{D}_{\text{at}}$. The matrix $\boldsymbol{D}_{\text{ta}}$ is given by $\boldsymbol{D}_{\text{ta}} = \boldsymbol{D}_{\text{at}}^T$.

D. The estimate of $\boldsymbol{D}_{tt}$ using low-rank matrix completion

After obtaining $\boldsymbol{D}_{\text{at}}$ and $\boldsymbol{D}_{\text{ta}}$, $\boldsymbol{D}_{\text{tt}}$ is the rest part of $\boldsymbol{D}$ to be determined. Supposing Ω denotes the set of all entries of $\boldsymbol{D}_{\text{aa}}$, $\boldsymbol{D}_{\text{at}}$ and $\boldsymbol{D}_{\text{ta}}$. The distance matrix recovering problem in (1) can be considered as a low-rank matrix completion (LRMC) problem [11]. It has been proven that the rank of matrix $\boldsymbol{D}$ is 4 for 2D geometry localization case and it has a low-rank property. And $\boldsymbol{D}_{\text{tt}}$ can be fixed by $\boldsymbol{D}_{\text{tt}} = \boldsymbol{D}_{\text{ta}}\boldsymbol{D}_{\text{aa}}^{\dagger}\boldsymbol{D}_{\text{at}}$, where $\boldsymbol{D}_{\text{aa}}^{\dagger}$ denotes pseudoinverse of $\boldsymbol{D}_{\text{aa}}$. The solver performs well with noiseless $\boldsymbol{D}$ and exactly known $\boldsymbol{D}_{\text{at}}$ which are impossible in practice.

The LRMC problem in (1) can be solved by formulating the problem into a rank minimization problem [11–13]. Unfortunately, it is non-convex and NP-hard. Candes [11] creatively showed that the rank minimization problem can be approximately solved by its convex relaxation problem

$$\begin{aligned} \text{minimize} \quad & \|\hat{\boldsymbol{D}}\|_* \\ \text{subject to} \quad & \hat{\boldsymbol{D}}(i,j) = \boldsymbol{D}(i,j), (i,j) \in \Omega, \end{aligned} \tag{11}$$

where $\|\cdot\|_*$ represents the Nuclear norm and the number of Ω obeys

$$m \geq 2r(M+N) - r^2, \tag{12}$$

where r is the rank of $\boldsymbol{D}$ and $\hat{\boldsymbol{D}}$ is the estimate of $\boldsymbol{D}$. The minimum value of M with respect to N should satisfy (12).

Note that $\boldsymbol{D}$ is not a positive semidefinite (PSD) matrix , we can not use LRMC method to recover $\boldsymbol{D}_{\text{tt}}$ directly. Inspired by the Lemma 1 in [13], we can construct a PSD matrix by the interior point method as

$$\begin{bmatrix} \boldsymbol{Y} & \hat{\boldsymbol{D}} \\ \hat{\boldsymbol{D}}^T & \boldsymbol{Z} \end{bmatrix} \geq 0. \tag{13}$$

Then the rank minimization problem in (11) can be equivalently reformulated as

$$\begin{aligned} \text{minimize} \quad & \delta \\ \text{subject to} \quad & \text{Tr}(\boldsymbol{Y}) + \text{Tr}(\boldsymbol{Z}) \leq 2\delta \\ & \begin{bmatrix} \boldsymbol{Y} & \hat{\boldsymbol{D}} \\ \hat{\boldsymbol{D}}^T & \boldsymbol{Z} \end{bmatrix} \geq \boldsymbol{0} \\ & \hat{\boldsymbol{D}}(i,j) = \boldsymbol{D}(i,j), (i,j) \in \Omega, \end{aligned} \tag{14}$$

where δ is a tolerance variable, $\boldsymbol{Y} \in \mathbb{R}^{(M+N)\times(M+N)}$ and $\boldsymbol{Z} \in \mathbb{R}^{(M+N)\times(M+N)}$ are the slack matrices, $\text{Tr}(\cdot)$ denotes the trace operator.

Furthermore, considering that $\boldsymbol{D}$ is a symmetric matrix, the problem in (14) can be simplified as

$$\begin{aligned} \text{minimize} \quad & \delta \\ \text{subject to} \quad & \text{Tr}(\boldsymbol{Q}) \leq \delta \\ & \begin{bmatrix} \boldsymbol{Y} & \hat{\boldsymbol{D}} \\ \hat{\boldsymbol{D}}^T & \boldsymbol{Y} \end{bmatrix} \geq \boldsymbol{0} \\ & \hat{\boldsymbol{D}}(i,j) = \boldsymbol{D}(i,j), (i,j) \in \Omega. \end{aligned} \tag{15}$$

Obviously, it is a semidefinite program problem which can be efficiently solved by CVX [14]. $\hat{\boldsymbol{D}}$ can be estimated by the modified interior method above and $\boldsymbol{D}_{\text{tt}}$ is given by $\hat{\boldsymbol{D}}(M+1 : M+N, N+1 : M+N)$.

E. The estimate of target locations using SDP relaxation

From the geometry relationship, we have

$$\left.\begin{aligned} \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2^2 &= \hat{d}_{i,j}^2 \\ \|\boldsymbol{o}_k - \boldsymbol{x}_j\|_2^2 &= \hat{d}_{k,j}^2 \end{aligned}\right\}, \tag{16}$$

where $i, j = 1, 2, \cdots, N$ and $k = 1, 2, \cdots, M$. Assuming a symmetric matric $\boldsymbol{H}$, and $\boldsymbol{H} = \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}}$, (16) can be rewritten as

$$\left.\begin{aligned} \boldsymbol{e}_{i,j}^T \boldsymbol{H} \boldsymbol{e}_{i,j} &= \hat{d}_{i,j}^2 \\ (\boldsymbol{o}_k; \boldsymbol{e}_j)^T \begin{pmatrix} \boldsymbol{I}_d & \hat{\boldsymbol{X}} \\ \hat{\boldsymbol{X}}^T & \boldsymbol{H} \end{pmatrix} (\boldsymbol{o}_k; \boldsymbol{e}_j) &= \hat{d}_{k,j}^2 \end{aligned}\right\}, \tag{17}$$

where $\boldsymbol{e}_{i,j}$ is a vector with 1 at the ith position,-1 at the jth position and zero everywhere else. $\boldsymbol{e}_j$ is the vector of all zeros except an -1 at the jth position.

The SDP method is to relax $\boldsymbol{H} = \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}}$ to $\boldsymbol{H} \succeq \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}}$, and $\boldsymbol{H} \succeq \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}}$ indicates that $\boldsymbol{H} - \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}} \succeq \boldsymbol{0}$. Fazel in [13] proves that the condition $\boldsymbol{H} - \hat{\boldsymbol{X}}^T\hat{\boldsymbol{X}} \succeq \boldsymbol{0}$ is equivalent to $\boldsymbol{W} = \begin{pmatrix} \boldsymbol{I}_d & \hat{\boldsymbol{X}} \\ \hat{\boldsymbol{X}}^T & \boldsymbol{H} \end{pmatrix} \geq \boldsymbol{0}$. And the matrix $\boldsymbol{W}$ can be computed by reformulating the problem in (17) as a standard SDP program as follows,

$$\begin{aligned} \text{minimize} \quad & \delta_1 + \delta_2 \\ \text{subject to} \quad & \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2^2 - \hat{d}_{i,j}^2 \leq \delta_1, \\ & \|\boldsymbol{x}_j - \boldsymbol{o}_k\|_2^2 - \hat{d}_{j,k}^2 \leq \delta_2, \\ & \boldsymbol{W} \succeq \boldsymbol{0}. \end{aligned} \tag{18}$$

The location estimates of target nodes $\hat{\boldsymbol{X}}$ can be easily obtained from above optimization problem.

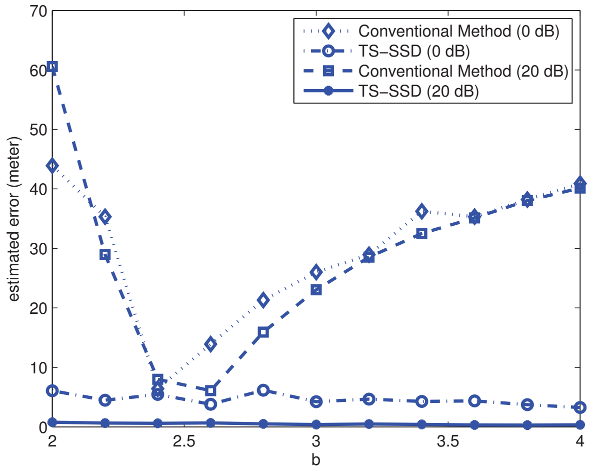

Fig. 1. Estimated error of $\boldsymbol{D}_{\text{at}}$ using TS-SSD and conventional methods versus different SNRs and β.

II. Numerical Results

Firstly, we investigate the computing accuracy of the $\boldsymbol{D}_{\text{at}}$. Consider a 2-D geometry of $M = 4$ anchors with known coordinates at $(-20, 20)$, $(20, 20)$, $(20, -20)$, $(-20, -20)$, while the unknown positions of three target nodes are $(7.5, 7.5)$, $(3, 7)$, $(15, 15)$. We generate the noisy RSSI values $P_{i,j}$ using (2) with $\beta = 2.5$, $d_0 = 1\text{m}$ and the noise variance, $\sigma_{n_i}^2$, is assigned with SNR $= -10\log_{10}\sigma_{n_i}^2$. In the conventional method [4], the element of $\hat{\boldsymbol{D}}_{\text{at}}$ is calculated by $\hat{d}_{i,j} = d_0 \cdot 10^{\frac{(P_0+n_i-P_{i,j})}{10\beta}}$, while TS-SSD estimates $\hat{\boldsymbol{D}}_{\text{at}}$ using (10). The estimated error is calculated by Error $= \left\|\hat{\boldsymbol{D}}_{\text{at}} - \boldsymbol{D}_{\text{at}}\right\|_{\text{F}}^2$. The errors of the conventional method and our method versus different β and SNRs are shown in Fig. 1. Here SNR=20 dB and 0 dB cases are considered and β varies from 2 to 4. From the figure, we find that TS-SSD method shows more robustness when β and SNRs change because it obtains $\hat{\boldsymbol{D}}_{\text{at}}$ using two stages differences technique and TLS solution. Secondly, we consider three methods for fixing $\boldsymbol{D}_{\text{tt}}$, namely, Singular Value Thresholding (SVT) [12], exact completion solver (ECS) [11], and our proposed method. The recovered errors were reported in Fig. 2. Among them, ECS is a closed form solution, which is derived in noiseless case. And it is very sensitive to noise level. SVT is an iterative solution for handling the case with noised $\boldsymbol{D}_{\text{tt}}$. And the performance of

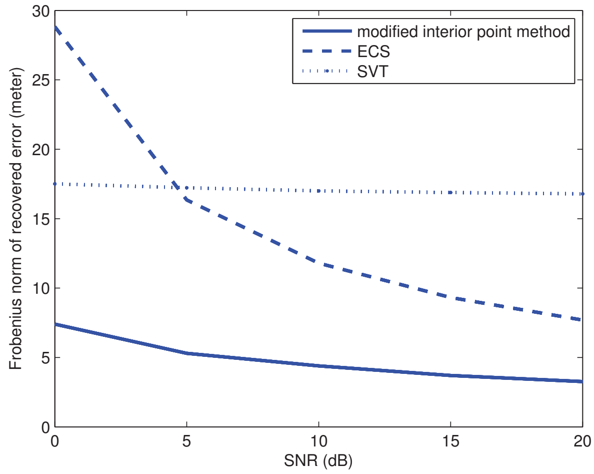

Fig. 2. The recovered error of $\boldsymbol{D}_{\mathrm{tt}}$ versus different SNRs using three different recovery methods.

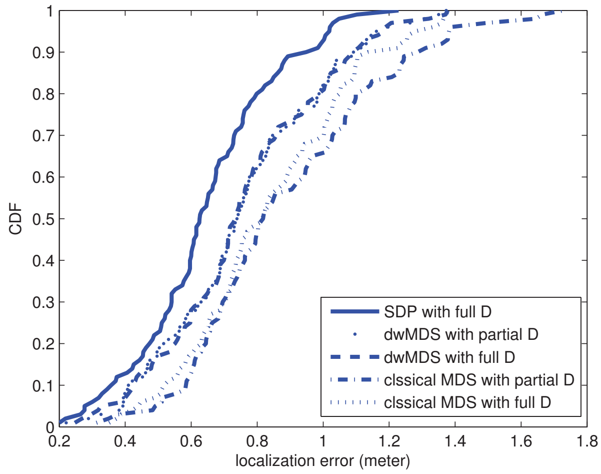

Fig. 3. Cumulative distribution function (CDF) versus localization errors of different methods (SNR = 0 dB).

TABLE I
THE AVERAGE RUN TIME OF DIFFERENT METHODS.

Methods	The proposed method	dwMDS	classical MDS
Run time (s)	0.5366	0.5193	0.0049

SVT is constrained by several parameters, including step size, number of iterations to achieve convergence, etc. So it has heavy computational burden with low accuracy. The results show that the performance of our proposed method is superior with respect to the other techniques in all listed SNR cases.

Thirdly, the cumulative distribution functions (CDF) of different localization methods, including our proposed SDP method, classical MDS [8], and dwMDS [9], are shown in Fig. 3. We consider that these methods in two cases: $\boldsymbol{D}$ is full recovered and it is not partial recovered. All results are obtained by averaging over 200 independent trials with SNR $= 0$ dB. It is worth observing that our method obtains better performance in the same noise level and it shows more robust than others as the value of SNR decreases. Therefore, our proposed method has good performance in accuracy and robustness.

All simulations were run on a Window 7 desktop computer with a 2.66 GHz Intel Core-i5 Quadcore CPU and 4 GB of RAM. The average run time of these methods are listed in Table I. We find that the complexity of our method is close to that of dwMDS and both of them are higher than classical MDS.

III. Conclusion

We address a TS-SSD localization algorithm using SDP relaxation. Based on the conventional path loss model , we derive a new linear model using twice difference techniques. A TLS solution and a modified interior point method are used to estimate the unknown distance matrix. Based on the estimated distance matrix, a localization approach using SDP relaxation is derived to obtain the position estimation of target nodes. The proposed method outperforms some existing methods in robustness, model error, and noise level. The simulation results verify the superiority of our proposed method.

Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grant (61201277).

References

[1] L. Lin, H. So, and Y. Chan, "Accurate and simple source localization using differential received signal strength," *Digital Signal Processing*, vol. 23, no. 3, pp. 736–743, 2013.

[2] L. Lin, H. So, F. K. Chan, Y. Chan, and K. Ho, "A new constrained weighted least squares algorithm for TDOA-based localization," *Signal Processing*, vol. 93, no. 11, pp. 2872–2878, 2013.

[3] R. M. Vaghefi, M. R. Gholami, and E. G. Strom, "RSS-based sensor localization with unknown transmit power," in *Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on*. IEEE, 2011, pp. 2480–2483.

[4] N. Salman, M. Ghogho, and A. H. Kemp, "On the joint estimation of the RSS-based location and path-loss exponent," *Wireless Communications Letters, IEEE*, vol. 1, no. 1, pp. 34–37, 2012.

[5] Y. Xu, J. Zhou, and P. Zhang, "RSS-based source localization when path-loss model parameters are unknown," *Communications Letters, IEEE*, vol. 18, no. 6, pp. 1055–1058, 2014.

[6] Y. Jin, W.-S. Soh, and W.-C. Wong, "Error analysis for fingerprint-based localization," *Communications Letters, IEEE*, vol. 14, no. 5, pp. 393–395, 2010.

[7] A. Mahtab Hossain, Y. Jin, W.-S. Soh, and H. N. Van, "SSD: A robust RF location fingerprint addressing mo-

bile devices' heterogeneity," *Mobile Computing, IEEE Transactions on*, vol. 12, no. 1, pp. 65–77, 2013.
[8] K. W. Cheung and S. H. C., "A multidimensional scaling framework for mobile location using time-of-arrival measurements," *Signal Processing, IEEE Transactions on,*, vol. 53, no. 2, pp. 460–470, 2005.
[9] J. A. Costa, N. Patwari, and A. O. Hero III, "Distributed weighted-multidimensional scaling for node localization in sensor networks," *ACM Transactions on Sensor Networks (TOSN)*, vol. 2, no. 1, pp. 39–64, 2006.
[10] J. A. Cadzow, "Spectral estimation: an overdetermined rational model equation approach," *Proceedings of the IEEE*, vol. 70, no. 9, pp. 907–939, 1982.
[11] E. J. Candès and B. Recht, "Exact matrix completion via convex optimization," *Foundations of Computational mathematics*, vol. 9, no. 6, pp. 717–772, 2009.
[12] J.-F. Cai, E. J. Candès, and Z. Shen, "A singular value thresholding algorithm for matrix completion," *SIAM Journal on Optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.
[13] M. Fazel, "Matrix rank minimization with applications," Ph.D. dissertation, PhD thesis, Stanford University, 2002.
[14] M. Grant, S. Boyd, and Y. Ye, "CVX: Matlab software for disciplined convex programming," 2008.